27.04.2026

İzmir Institute of Technology
MSE 222 Applied Mathematics for Materials Science and Engineering, Spring 2026
Quiz II - Solution Key

Name: ____________________

Student ID: ____________________

Duration: 40 Minutes

Grade Table

Question:	1	2	3	Total
Points:	4	4	4	12
Score:				

1. (4 points) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that first reflects points across the y-axis, then stretches them vertically by a factor of 2 and then rotates them counterclockwise by $\frac{\pi}{2}$ radians. Find the standard matrix A for T.

   Reflection across the y-axis is represented by
   $$A_1 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}.$$
   Second, vertical stretching by a factor of 2 is represented by
   $$A_2 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}.$$
   Third, counterclockwise rotation by $\frac{\pi}{2}$ radians is represented by
   $$A_3 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$
   Since the transformations are applied in the order
   $$\text{reflection} \longrightarrow \text{vertical stretch} \longrightarrow \text{rotation},$$
   the standard matrix of T is
   $$A = A_3A_2A_1 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ -1 & 0 \end{bmatrix}.$$

2. Let
   $$A = \begin{bmatrix} -1 & -3 \\ -2 & -3 \end{bmatrix}.$$
   (a) (1 point) Find the determinant of A.

   $\det(A) = (-1)(-3) - (-3)(-2) = 3 - 6 = -3.$

   (b) (1 point) Find the matrix of cofactors of A.
   The matrix of cofactors of A is
   $$C = \begin{bmatrix} -3 & 2 \\ 3 & -1 \end{bmatrix}.$$

(c) (1 point) Find the adjoint of A.

The adjoint of A is the transpose of the cofactor matrix. Hence,

$$\operatorname{adj}(A) = \begin{bmatrix} -3 & 3 \\ 2 & -1 \end{bmatrix}.$$

(d) (1 point) Find the inverse of A.

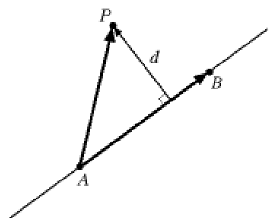
Using

$$A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A),$$

we get

$$A^{-1} = -\frac{1}{3} \begin{bmatrix} -3 & 3 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -\frac{2}{3} & \frac{1}{3} \end{bmatrix}.$$

3. (4 points) The distance d of a point P to the line through points A and B is the length of the component of $\overline{AP}$ that is orthogonal to $\overline{AB}$, as indicated in the diagram.



Find the distance from $P = (-5, -4, 5)$ to the line through the points $A = (0, -2, 4)$ and $B = (-4, 3, -2)$.

We have

$$\overrightarrow{AB} = \begin{bmatrix} -4 \\ 5 \\ -6 \end{bmatrix}$$

and

$$\overrightarrow{AP} = \begin{bmatrix} -5 \\ -2 \\ 1 \end{bmatrix}$$

The scalar projection of $\overrightarrow{AP}$ onto $\overrightarrow{AB}$ is

$$\operatorname{comp}_{\overrightarrow{AB}} \overrightarrow{AP} = \frac{\overrightarrow{AP} \cdot \overrightarrow{AB}}{\|\overrightarrow{AB}\|} = \frac{4}{\sqrt{77}}.$$

Since the distance is the length of the component of $\overrightarrow{AP}$ orthogonal to $\overrightarrow{AB}$, we get

$$d^2 = \|\overrightarrow{AP}\|^2 - \left(\operatorname{comp}_{\overrightarrow{AB}} \overrightarrow{AP}\right)^2.$$

Thus,

$$d^2 = 30 - \left(\frac{4}{\sqrt{77}}\right)^2 = 30 - \frac{16}{77} = \frac{2294}{77}.$$

Therefore,

$$\boxed{d = \sqrt{\frac{2294}{77}}}.$$

1. Find the Fourier series of the function f defined on $[-\pi, \pi]$ by
$$f(x) = \begin{cases} 0, & -\pi \le x < 0, \\ x, & 0 \le x \le \pi. \end{cases}$$

2. Find the Fourier series of the function f defined on $[-\pi, \pi]$ by
$$f(x) = |x|.$$

3. Find the Fourier series of the function f defined on $[-\pi, \pi]$ by
$$f(x) = \begin{cases} -1, & -\pi \le x < 0, \\ 1, & 0 < x \le \pi. \end{cases}$$