16.03.2026

İzmir Institute of Technology
MSE 222 Applied Mathematics for Materials Science and Engineering, Spring 2026
Quiz I - Solution Key

Name: ______________________________

Student ID: ______________________________

Duration: 40 Minutes

Grade Table

Question:	1	2	3	4	Total
Points:	3	3	4	2	12
Score:					

1. (3 points)

$$A = \left[\begin{array}{cccc|c} 1 & 0 & 4 & 0 & 4 \\ 0 & 1 & 2 & 0 & -4 \\ 0 & 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

(a) Is the matrix in row echelon form? Yes.

(b) Is the matrix in reduced row echelon form? Yes.

(c) If this matrix were the augmented matrix for a system of linear equations, would the system be inconsistent, dependent, or independent? Dependent.

2. Consider the following Gauss-Jordan reduction:

$$\underbrace{\begin{bmatrix} 3 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{A} \xrightarrow{E_1} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E_1 A} \xrightarrow{E_2} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E_2 E_1 A = I} = I.$$

(a) (2 points) Find E_1, E_2.

$$E_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(b) (1 point) Write A as a product $A = E_1^{-1} E_2^{-1}$ of elementary matrices:

$$A = E_1^{-1} E_2^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

3. (a) (2 points) Find the inverse of the matrix.

$$A = \begin{bmatrix} 4 & -12 & 11 \\ -2 & 7 & -7 \\ 1 & -3 & 3 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 4 & -12 & 11 & 1 & 0 & 0 \\ -2 & 7 & -7 & 0 & 1 & 0 \\ 1 & -3 & 3 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3 \leftrightarrow R_1} \left[\begin{array}{ccc|ccc} 1 & -3 & 3 & 0 & 0 & 1 \\ -2 & 7 & -7 & 0 & 1 & 0 \\ 4 & -12 & 11 & 1 & 0 & 0 \end{array}\right]$$

$$\xrightarrow[R_3-4R_1 \to R_3]{R_2+2R_1 \to R_2} \left[\begin{array}{ccc|ccc} 1 & -3 & 3 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 1 & 2 \\ 0 & 0 & -1 & 1 & 0 & -4 \end{array}\right] \xrightarrow{-R_3 \to R_3} \left[\begin{array}{ccc|ccc} 1 & -3 & 3 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 1 & 2 \\ 0 & 0 & 1 & -1 & 0 & 4 \end{array}\right]$$

$$\xrightarrow[R_2+R_3 \to R_2]{R_1-3R_3 \to R_1} \left[\begin{array}{ccc|ccc} 1 & -3 & 0 & 3 & 0 & -11 \\ 0 & 1 & 0 & -1 & 1 & 6 \\ 0 & 0 & 1 & -1 & 0 & 4 \end{array}\right] \xrightarrow{R_1+3R_2 \to R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 3 & 7 \\ 0 & 1 & 0 & -1 & 1 & 6 \\ 0 & 0 & 1 & -1 & 0 & 4 \end{array}\right]$$

Hence,

$$\boxed{A^{-1} = \begin{bmatrix} 0 & 3 & 7 \\ -1 & 1 & 6 \\ -1 & 0 & 4 \end{bmatrix}.}$$

(b) (2 points) Use the answer from part (a) to solve the linear system

$$\begin{cases} 4x_1 - 12x_2 + 11x_3 = -1, \\ -2x_1 + 7x_2 - 7x_3 = 5, \\ x_1 - 3x_2 + 3x_3 = -2. \end{cases}$$

The given system can be written in the matrix form

$$A\mathbf{x} = \mathbf{b},$$

where

$$A = \begin{bmatrix} 4 & -12 & 11 \\ -2 & 7 & -7 \\ 1 & -3 & 3 \end{bmatrix}, \qquad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} -1 \\ 5 \\ -2 \end{bmatrix}.$$

Since we have already computed the inverse matrix A^{-1}, we multiply both sides of $A\mathbf{x} = \mathbf{b}$ by A^{-1} from the left to write $A^{-1}A\mathbf{x} = A^{-1}\mathbf{b}$. Hence,

$$\mathbf{x} = A^{-1}\mathbf{b}.$$

Using

$$A^{-1} = \begin{bmatrix} 0 & 3 & 7 \\ -1 & 1 & 6 \\ -1 & 0 & 4 \end{bmatrix},$$

we obtain

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 3 & 7 \\ -1 & 1 & 6 \\ -1 & 0 & 4 \end{bmatrix} \begin{bmatrix} -1 \\ 5 \\ -2 \end{bmatrix}.$$

Now compute the matrix-vector product:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0(-1) + 3(5) + 7(-2) \\ (-1)(-1) + 1(5) + 6(-2) \\ (-1)(-1) + 0(5) + 4(-2) \end{bmatrix} = \begin{bmatrix} 1 \\ -6 \\ -7 \end{bmatrix}.$$

Therefore, the solution of the system is

$$\boxed{x_1 = 1, \qquad x_2 = -6, \qquad x_3 = -7.}$$

4. (2 points) Express the vector $\vec{v} = \begin{bmatrix} 8 \\ 17 \end{bmatrix}$ as a linear combination of $\vec{x} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ and $\vec{y} = \begin{bmatrix} -1 \\ -3 \end{bmatrix}$.

We seek scalars a and b such that

$$\vec{v} = a\vec{x} + b\vec{y} = a\begin{bmatrix} 4 \\ 5 \end{bmatrix} + b\begin{bmatrix} -1 \\ -3 \end{bmatrix}.$$

This gives the system

$$\begin{bmatrix} 8 \\ 17 \end{bmatrix} = \begin{bmatrix} 4a - b \\ 5a - 3b \end{bmatrix},$$

so that

$$\begin{cases} 4a - b = 8, \\ 5a - 3b = 17. \end{cases}$$

From the first equation,

$$b = 4a - 8.$$

Substituting this into the second equation, we get

$$5a - 3(4a - 8) = 17.$$

Simplifying,

$$5a - 12a + 24 = 17,$$
$$-7a = -7,$$
$$a = 1.$$

Then

$$b = 4(1) - 8 = -4.$$

Therefore,

$$\vec{v} = 1\vec{x} - 4\vec{y}.$$

Hence, the required linear combination is

$$\boxed{\begin{bmatrix} 8 \\ 17 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix} - 4\begin{bmatrix} -1 \\ -3 \end{bmatrix}.}$$